DIFFRACTION OF AN ELASTIC WAVE IN A DISC

A. M. Skobeev*

UDC 534.222:539.3

A numerical solution is constructed for the axisymmetric problem of the diffraction of a plane longitudinal wave in a rigid disc (cylinder) of finite thickness. The disc is enclosed in an unbounded elastic medium; at the contact surface, the tangential stresses are limited by some constant. The incident wave moves along the axis of the cylinder and has the form of a semiinfinite washed-out step. At the same time, a solution is obtained to the corresponding static problem. A study was made of the dependence of the rate of motion of the cylinder and the stress field on the parameters of the problem. In particular, it is shown that the contact conditions have a considerable effect on the stress field only near the lateral surface. The results obtained can be useful for evaluating the errors in measurement of the stresses and velocities in an elastic medium, and possibly also in certain other cases.

1. Within the framework of the dynamic theory of elasticity, this paper considers the axisymmetric problem of the interaction between a longitudinal wave and a rigid cylinder of finite dimensions. The cylindrical coordinates z and r are used; the z axis coincides with the axis of the cylinder, which occupies the region $-H/2 \leq z \leq H/2$, $r \leq 1$ (Fig. 1). The region outside of the cylinder is filled by an elastic medium. The measurement units were chosen so that the radius of the cylinder, the density of the medium, and the rate of propagation of longitudinal waves in the medium were equal to unity.

In what follows, the following basic designations will be used: ρ is the density of the cylinder; μ is the shear modulus of the medium; t is the time; u(t, r, z) and w(t, r, z) are displacements along r and z; v(t) is the velocity; $w_0(t)$ is the displacement of the cylinder; σ_{zz}, $\tau = \sigma_{rz}$ are the components of the stress tensor, connected with the displacements by Hooke's law

$$\sigma_{zz} = w_z + (1-2\mu)(u_r + u/r), \quad \tau = \mu(u_z + w_r) \quad (1.1) \tag{1.1}$$

In addition, the following auxiliary designations are introduced: Γ is the contour of the cylinder in the coordinates r, z; Q_1 and Q_2 are the displacements of the cylinder along the normal and the tangent to Γ; $Q_1 = 0$, $Q_2 = w_0$ at the lateral surface; $Q_1 = w_0$, $Q_2 = 0$ at the bases; q_1 and q_2 are the displacements in the medium at the boundary with the cylinder, q_1 along a normal to Γ, q_2 along the tangent; a dot denotes a partial derivative with respect to t.

Outside of the cylinder, the dynamic equations of the theory of elasticity are satisfied for u and w

$$\begin{aligned} u_{tt} &= \mu u_{zz} + u_{rr} + (1-\mu) w_{rz} + u_z / r - u / r^2 \\ w_{tt} &= w_{zz} + \mu w_{rr} + \mu w_r / r + (1-\mu)(u_{zr} + u_z / r) \end{aligned} \tag{1.2}$$

The equation of motion of the cylinder is

$$\rho \frac{H}{2} \frac{d^2 w_0}{dt^2} = \int_\Gamma r\sigma_{zz}\, dr + \tau\, dz \tag{1.3}$$

Equations (1.2) and (1.3) represent a boundary-value problem with initial conditions at t = 0 and boundary conditions at Γ.

*Deceased.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Teknicheskoi Fiziki, No. 3, pp. 139-150, May-June, 1972. Original article submitted February 7, 1972.

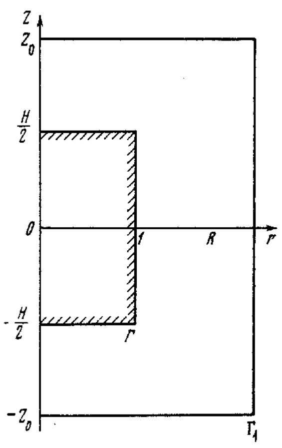

Fig. 1

The initial conditions describe a plane longitudinal wave, falling from infinity on the upper base of the cylinder. Ahead of the leading front of the wave, the medium is quiescent and not loaded; behind the front, there is a monoaxial deformation with $\sigma_{zz} = 1$. These conditions have the form

$$w_0(0) = 0, \;\; v(0) = 0, \;\; u(0, r, z) = u_t(0, r, z) = 0 \qquad (1.4)$$
$$w_z(0, r, z) = w_t(0, r, z) = f((H + T - 2z) / T)$$

Here

$$f(\xi) = 0 \;\text{ at }\; \xi \geqslant 1, \; f(\zeta) = 1 \;\text{ at }\; \xi \leqslant -1, \; f(\zeta) = (1 + \xi^2 \operatorname{sign} \xi) / 2 - \xi \;\text{ at }\; -1 < \xi < 1 \qquad (1.5)$$

The parameter T, entering into (1.4), characterizes the degree of washing-out of the wave.

At the surface of the cylinder, there are two boundary conditions. The first condition arises from the assumption that the cylinder is rigid. It has the form $q_1 = Q_1$ at Γ.

The second condition corresponds to a simplified law of dry friction

$$\dot{q}_2 = \dot{Q}_2, \quad \text{если} \;\; |\tau| < k$$
$$\tau = k \operatorname{sign}(\dot{q}_2 - \dot{Q}_2), \quad \text{if} \;\; |\tau| = k \qquad (1.6)$$

These conditions differ from Coulomb's law in that the quantity k introduced into (1.6) characterizing the adhesion of the medium to the surface, does not depend on the normal stress.

It must be noted that, if $k = 0$, from (1.6), it follows that $\tau = 0$, which corresponds to slippage conditions. At sufficiently large values of k, $\dot{q}_2 = \dot{Q}_2$, which corresponds to adhesion conditions. In these important partial cases, the problem becomes linear.

The problem formulated arises naturally with the study of the effect of shock loading on a body containing a rigid cylindrical inclusion. It is clear that, depending on the situation, different data with respect to the phenomenon may be necessary; it is therefore, expedient to fix the physical interpretation of the problem.

In what follows, it will be assumed that the cylinder is a stress pickup, enclosed in an unbounded elastic medium. The sensing element occupies a certain part of the upper base of the cylinder and does not affect the stress field. It is assumed that the measured stress lies between the maximal and minimal normal stresses acting on the sensing element. The object of the measurement is, obviously to obtain information on the stress in the wave; the main interest lies in the difference between the measured stress and the stress in the incident wave. Therefore, in what follows, the principal emphasis will be on a study of the effect of the parameters of the process on the distribution of the normal stresses at the upper base of the cylinder.

2. The formulated problem was solved numerically. The equations of motion and the boundary conditions were replaced by finite-difference relationships, and the system of equations obtained was solved on a BÉSM-3SM digital computer. It is clear that, under these circumstances, a solution may be obtained only in a finite region. A perturbation, brought about by the motion of the cylinder, for finite values of t, is propagated to a finite region, outside of which the solution has the form

$$u = 0, \;\; w = f(t - z + H / 2) \qquad (2.1)$$

It appears natural to calculate the solution in this region; however, the technical characteristics of the machine did not permit using this method. Therefore, additional boundary conditions were introduced with $z = \pm z_0$ and $0 \leq r \leq R$, $r = R$ and $-z_0 \leq z \leq z_0$, simulating conditions at infinity. It was assumed that, at the boundary introduced, the perturbed motion is close to one-dimensional. The boundary introduced is designated Γ_1 (Fig. 1). For the normal and tangential displacements of the perturbed motion toward the new boundary, the previous designations q_1 and q_2 are used; the normal to the boundary is denoted by l. If we define $C_1 = 1$, $C_2 = \sqrt{\mu}$, then, at Γ_1, the one-dimensional equations of the theory of elasticity will be satisfied

$$q_{\alpha tt} = C_\alpha^2 q_{\alpha ll}, \quad \alpha = 1,2 \qquad (2.2)$$

In (2.2), there is no summation with respect to α; the subscript l denotes the derivative along the normal to Γ_1 and $q_1 = w - f(t - z + H/2)$, $q_2 = u$ at the upper and lower sections; $q_1 = u$, $q_2 = w - f(t - z + H/2)$ at the lateral section.

The general solution of any of Eqs. (2.2) consists of the sum of two arbitrary functions, one of which describes a wave going away toward infinity, and the other a wave coming from infinity. Since the perturbed motion contains only waves of the first type, the boundary conditions must intersect waves of the second type. Such boundary conditions are the following analog of the Sommerfeld principle of radiation:

$$q_{\alpha t} + C_\alpha q_{\alpha l} = 0 \ \text{ at } \Gamma_1 \quad (\alpha = 1, 2) \tag{2.3}$$

At $\rho = 0$, Eq. (1.3) degenerates; therefore, using (1.1) and (1.2), we transform it to the form

$$\rho \frac{H}{2} \frac{d^2 w_0}{dt^2} + \iint\limits_{D_1} w_{tt} r \, dr \, dz = \int\limits_{\Gamma_2} r w_z \, dr + \mu r \, w_r \, dz + (1 + \mu)(1 + h/2)(u_a - u_b) \tag{2.4}$$

Here, h is a constant; Γ_2 is a contour, made up of the straight lines

$$-(H + h)/2 \leqslant z \leqslant (H + h)/2, \qquad r = 1 + h/2,$$
$$z = \pm (H + h)/2, \quad 0 \leqslant r \leqslant 1 + h/2$$

where D_1 is the region included between Γ and Γ_2; u_a and u_b are the values of u at the upper and lower corners of Γ_2.

In a cylindrical system of coordinates, there arise conditions at $r = 0$ which, from considerations of symmetry, are taken in the form

$$u = 0, \ w_r = 0$$

The initial conditions were set not at $t = 0$, but at $t = H/2 - z_0 < 0$. This comes down to a situation in which the sought functions and their derivatives are equal to zero at $t = H/2 - z_0$ and $z \le z_0$.

Thus, the system of equations (1.2) is solved with the boundary conditions (1.6), (2.3), (2.4) and null initial conditions.

As is usual, the region included between Γ and Γ_1 is divided by straight lines parallel to the coordinate axes into squares with the side h. All the functions are calculated only at the nodes of the grid obtained, and for discrete values of the time. It is assumed that Γ and Γ_2 pass through nodes of the grid. This means that $1/h$, H/h, z_0/h, and R/h are all whole numbers.

The solution is calculated for successive values of t, with the spacing t_0, starting from $t = H/2 - z_0 + t_0$.

In what follows, the arguments of the functions are not calculated; the solution for the moment of time t is called the middle solution, for $t - t_0$, it is called the lower solution, and for $t + t_0$, it is called the solution for the upper layer. To obtain an approximation of the derivatives at the external points of the region in the middle layer, the central difference operators δ_z, δ_{zz}, and δ_{zr} are introduced using the formulas

$$\delta_z f_{ij} = (f_{i,j+1} - f_{i,j-1}) / 2h \quad \delta_{zz} f_{ij} = (f_{j,j+1} - 2f_{i,j} + f_{i,j-1} / h^2 \quad \delta_{zr} f_{ij} = (f_{i+1,j+1} + f_{i-1,j-1} - f_{i,j+1} - f_{i-1,j} / 4h^2 \tag{2.5}$$

Here, $f(r, z)$ is an arbitrary function of r and z. Analogously, the operators δ_r, δ_{rr}, δ_l, δ_{ll}, δ_t, and δ_{tt} are introduced, except that in the expressions for the two last operators, t_0 is used instead of h. Thus, for the corresponding derivatives, the operators introduced yield an approximation of the second order.

Since Eqs. (1.2) are determined only at external points of the region, for them we can immediately write the difference analogies

$$\begin{gathered} \delta_{tt} u = \mu \delta_{zz} u + \delta_{rr} u + (1 - \mu) \delta_{rz} w + \delta_r u / (hi) - u / (hi)^2 \\ \delta_{tt} w = \delta_{zz} w + \mu \delta_{rr} w + \mu \delta_r w / (hi) + (1 - \mu) \delta_{rz} u + (1 - \mu) \delta_z u / (hi) \end{gathered} \tag{2.6}$$

The expressions obtained describe a three-layer explicit scheme of the second order of accuracy for the system of equations (1.2). For the scheme to be stable, the Courant criterion must be satisfied [1]; therefore, in what follows, it is assumed that $t_0 = h/2$.

The initial conditions for the system of equations (2.6) reduce to the fact that all the functions are equal to zero at $t = H/2 - z_0 - t_0$ and $t = H/2 - z_0$.

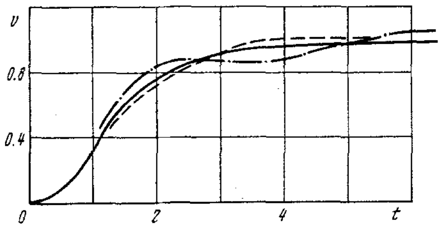

Fig. 2

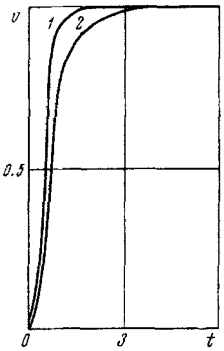

Fig. 3

The boundary conditions are of a more complex character. For the conditions at the external boundary, we can use the central difference relationships introduced into (2.5). The difference analogy of conditions (2.3) has the form (no summation with respect α; l is the normal to Γ_1)

$$\delta_t q_\alpha + C_\alpha \delta_l q_\alpha = 0 \text{ на } \Gamma_1 \quad (\alpha = 1, 2) \tag{2.7}$$

Since central difference relationships are used in (2.7), a point lying outside the region enters into (2.7). To exclude this point, we make use of the fact that, at the boundary, q_α also satisfies Eqs. (2.2). The difference approximation of these equations $\delta_{tt} q_\alpha = C_\alpha{}^2 \delta_{ll} q_\alpha$ contains the same point, lying outside of the region under consideration, as in (2.7). This permits excluding this point and obtaining an approximation of the boundary conditions in the form (q_α is taken in the middle layer)

$$2C_\alpha (\delta_t q_\alpha + C_\alpha \delta_l q_\alpha) / h + \delta_{tt} q_\alpha - C_\alpha{}^2 \delta_{ll} q_\alpha = 0 \tag{2.8}$$

Boundary condition (2.4) was also approximated by central difference relationships. The integral entering into the left-hand part is of the order h and can be transformed to the form

$$\iint\limits_{D_1} w_{tt} r \, dr \, dz = \frac{h}{2} \frac{d^2 w_0}{dt^2} + \frac{h}{2} \int\limits_{\Gamma_3} w_{tt} \, dz + O(h^2) \tag{2.9}$$

Here, Γ_3 is the segment $r = 1$, $-H/2 < z < H/2$.

The integrals with respect to Γ_2 and Γ_3 were calculated using the trapezoid rule, while the functions under the integral signs were taken at the points of intersection between Γ_2 and Γ_3, and the straight lines of the grid. The symbol Σ is used for the corresponding integral sums. Since Γ_2 does not pass through nodes of the grid, for an approximation of the first derivatives, the operators $\delta_z^{1/2}$ and $\delta_r^{1/2}$, analogous to those introduced into (2.5) were used, but with a spacing of h/2.

The difference analog of (2.4) has the form

$$\frac{(H+h)}{2} \delta_{tt} w_0 = \sum_{\Gamma_2} r (\delta_z^{1/2} w + \mu \delta_r^{1/2} w) + \frac{h}{2} \sum_{\Gamma_3} \delta_{tt} w + (1 - \mu)\left(1 + \frac{h}{2}\right) \frac{u(t, 1+h, h+H/2) - u(t, 1+h, -h-H/2)}{2} + O(h^2) \tag{2.10}$$

Here, $\delta_{tt} w$ was calculated in the lower layer. In the derivation of (2.10) use was made of the fact that u_a and u_b and the left-hand part of (2.9), have an order of magnitude h; thus, for them, an approximation of the first order is suitable.

For the remaining boundary conditions, there are required unilateral operators of the second order, determined in the second layer

$$\delta_z{}^1 f = (4f(z+h) - f(z+2h) - 3f(z)) / 2h \tag{2.11}$$

and the analogous operators δ_r^1, δ_l^1, and δ_t^1; only in the last operator is t_0 used instead of h.

At $r = 0$, the condition has the form

$$u = 0, \ \delta_r{}^1 w = 0$$

The condition at the boundary of the cylinder $q_1 = Q_1$ remains unchanged, while condition (1.6) assumes the form (l is the normal to Γ; $k_1 = k/\mu$)

$$\begin{aligned} &\delta_l{}^1 (q_2 - Q_2) = 0, \quad \text{if} \quad |\delta_l{}^1 q_2| < k_1 \\ &\delta_l{}^1 q_2 = k_1 \operatorname{sign} \delta_l{}^1 (q_2 - Q_2), \quad \text{if} \quad |\delta_l{}^1 q_2| = k_1 \end{aligned} \tag{2.12}$$

The conditions obtained constitute a system of nonlinear equations for determining q_2 at Γ. Each of the equations contains only one unknown; therefore, this system is easily solved. The solution is unique and has the form

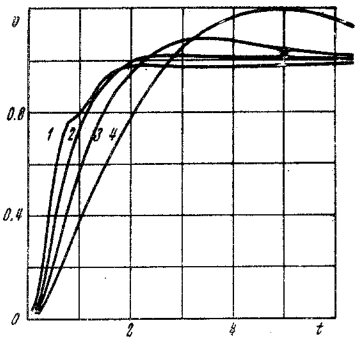

Fig. 4

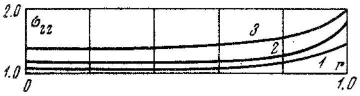

Fig. 5

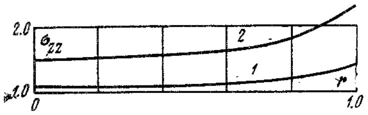

Fig. 6

$$q_2 = -y_1, \quad \text{if} \quad |y_1 + y_2| < k_2$$
$$q_2 - y_2 - k_2 \operatorname{sign}(y_1 + y_2), \quad \text{if} \quad |y_1 + y_2| \geqslant k_2 \tag{2.13}$$

Here

$$k_2 = 2hk_1 / \mu, \quad y_1 = 2h\delta_t^1 (q_2 - Q_2) / 3 - q_2, \quad y_2 = 2h\delta_t^1 q_2 / 3 + q_2$$

y_1 and y_2 do not contain q_2 at Γ.

Thus, the solution in the upper layer is completely determined. Moving from layer to layer, we can construct a solution for any given values of t.

3. The above scheme was put into practice in the form of a program with only slight changes. The calculation was carried out until steady-state conditions had been attained and took from 20 to 60 min; H was varied from 0.5 to 2.0, μ from 0.1 to 0.5, k from 0 to 1.0, ρ from 0.5 to 4.0, T from 0.5 to 1.0, z_0 from 2.5 to 5.0, R from 3.0 to 6.0, h from 0.1 to 0.2. Thus, only the case of large values of H remained uninvestigated. This gap is explained by the fact that, with a fixed spacing with respect to h and r, for large values of H, too many points of the grid are required.

As a result of the calculations it became clear that the most stable characteristic of the process is the velocity of the cylinder v(t), which is practically independent of μ and k, and depends only weakly on ρ and H. The solid line on Fig. 2 shows the dependence of v on t for $\mu = 0.1$, $k = 0$, $H = 2.0$, $\rho = 1.0$, $T = 0.25$, $h = 0.2$, $z_0 = 2.5$, $R = 8.0$. The dotted line corresponds to $\mu = 0.5$, $k = 1.0$, the dash-dot line to $\mu = 0.5$, $k = 0$, while the remaining parameters are the same. Calculations were carried out for intermediate cases ($\mu = 0.2, 0.3, 0.4$, and $k = 0$); the difference obtained was still less. Analogous results were also obtained for H = 0.5, 0.8, 1.0, and 1.6. Since in the theory of elasticity $\mu \leq 0.5$ [2], and at $k = 1.0$, no slippage was observed, it may be assumed that for $H \leq 2.0$ and $\rho = 1.0$, the velocity of motion of the cylinder is practically independent of μ and k.

Figures 3 and 4 show the dependence of v(t) on H and ρ. On Fig. 3, curves 1 and 2 correspond to H = 0.5, 1.0 and $\rho = 0.3$, $\rho = 1.0$, $k = 0$, $T = 0.5$; on Fig. 4, curves 1, 2, 3, 4 correspond to $\rho = 0.5, 1.0, 2.0, 4.0$ and $H = 1.0$, $\mu = 0.3$, $k = 0$, $T = 0.5$.

It can be seen that the dependence of the velocity on H and ρ is brought out clearly and that it corresponds to intuitive concepts.

The dependence of the stress field on the parameters of the problem comes out even more clearly. The essential point in the problem is the value of σ_{zz} at the surface of the cylinder. Figure 5 illustrates the value of $\sigma_{zz}(t_1, r, H/2)$; t_1 is chosen so that steady-state conditions have set in. Curve 1 on this figure shows σ_{zz} as a function of r for H = 0.5; curve 2 for H = 1.0; curve 3 for H = 2.0; the other parameters are

$$\mu = 0.3, \ \rho = 1.0, \ k = 0$$

On Fig. 6, curve 1 shows σ_{zz} for $\mu = 0.1$; curve 2 for $\mu = 0.5$; the other parameters are

$$H = 2.0, \ \rho = 1.0, \ k = 0$$

As an important special circumstance, it must be noted that σ_{zz} at the base of the cylinder is practically independent of k, while, at the same time, σ_{zz} depends strongly on k at the lateral surface. The dependence of σ_{zz} at the center of the upper base of the cylinder on k for $\mu = 0.1, 0.2, 0.3, 0.4, 0.5$ and $H = 2.0$, $t = \infty$, is shown on Table 1.

Figure 7 shows the dependence of σ_{zz} on z at the lateral surface of the cylinder for k = 0.1, 0.3 and H = 2.0, $\mu = 0.3$.

The dependence of σ_{zz} at the center of the upper base of the cylinder on μ and H for $k = 0.0$, $\rho = 1.0$, $t = \infty$, is shown on Table 2.

TABLE 1

k	μ				
	0.10	0.20	0.30	0.40	0.50
0.0	1.14	1.27	1.36	1.44	1.51
1.0	1.14	1.25	1.34	1.42	1.48

TABLE 2

H	μ				
	0.10	0.20	0.30	0.40	0.50
0.5	1.03	1.08	1.10	1.12	1.14
0.8	1.03	1.09	1.13	1.16	1.22
1.0	1.05	1.09	1.15	1.21	1.25
1.6	1.10	1.20	1.29	1.35	1.41
2.0	1.14	1.27	1.36	1.44	1.51

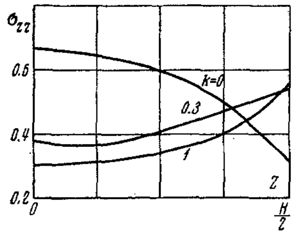

Fig. 7

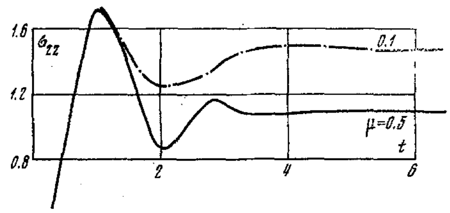

Fig. 8

Figure 8 gives the dependence $\sigma_{zz}(t)$ at the center of the upper base of the cylinder for $\mu = 0.1$, 0.5 and H = 2.0, ρ = 1.0, T = 0.25.

Figures 9-12 give the distributions of the stress $\sigma_{zz}(r, z, t)$ for moments of time t = 1.0, 1.5, 2.0, 5.0 and the following combination of parameters

$$\mu = 0.3,\ k = 1.0,\ \rho = 1.0,$$
$$H = 1.0,\ T = 0.5$$

These figures were obtained with the aid of a special program in which the read-out of an ATsPU-128 computer was used. The coordinate z is plotted along the axis of ordinates, and r along the axis of abscissas. The contour of the cylinder is indicated by the dashed line; the figures show only the region $|z| \le (H/2 + z_0)/2$, $0.5 \le r \le 1.5$, although the whole region $|z| \le z_0$, $0 \le r \le R$ was developed in the computer. The numbers 1, 2, 3, . . . denote zones of low values, and the letters high values of the stresses, compared to the stresses in the incident wave. The white fields, with the exception of the white field in front of the symbol 9 ahead of the front of the incident wave in Fig. 9, correspond to the stress in the incident wave. A transition to the following symbol corresponds to a change in σ_{zz} by 0.05. In particular, the absence of a footnote denotes $\sigma_{zz} = 1.0 \pm 0.025$, the symbol 1 denotes $\sigma_{zz} = 0.95 \pm 0.025$, and the symbol A denotes $\sigma_{zz} = 1.05 \pm 0.025$. In Fig. 9, the leading boundary of the wave front is located at the level of the lower base of the cylinder. In the central part of the upper base, the stress is $\sigma_{zz} = 1.10 \pm 0.025$, which, at r > 0.5, rises with approach to an angular point. Below the lower base, in the central part of the stress is $\sigma_{zz} = 0.90 \pm 0.025$. In the vicinity of an angular point, zones of stress concentration appear clearly (the symbol A). Along the lateral surface of the cylinder, the unstable character of the stress distribution is clearly marked. On the whole, the stresses here rise from the edges toward the lateral surface, where $\sigma_{zz} = 0.95 \pm 0.025$.

On Fig. 10, the stress distribution along the lateral surface becomes more symmetrical; however, the character of the distribution of the stresses, i.e., their increase from the edge toward the center, is retained. In the vicinity of angular points, above the upper and below the lower bases of the cylinder, there is an appreciable growth of the stress concentration. The stresses in the central part of the lower base rise ($\sigma_{zz} = 0.95 \pm 0.025$).

In Fig. 11, the stress distribution immediately along the lateral surface is close to steady-state conditions. Above the upper and below the lower bases, there is an increase in the zones of stress concentration.

Figure 12 corresponds to fully established quasi-steady-state conditions. The concentration of the stresses along the upper and lower bases is identical, and, at $0 \le r \le 0.5$–0.6, is equal to $\sigma_{zz} = 1.10 \pm 0.025$.

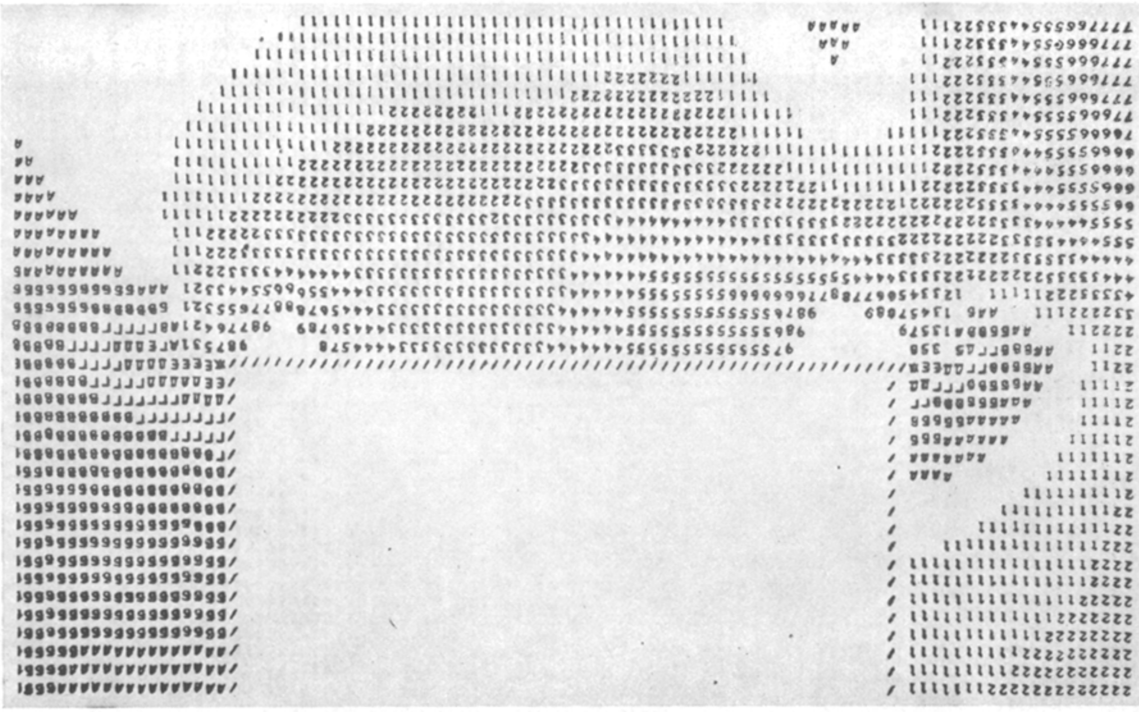

Fig. 10

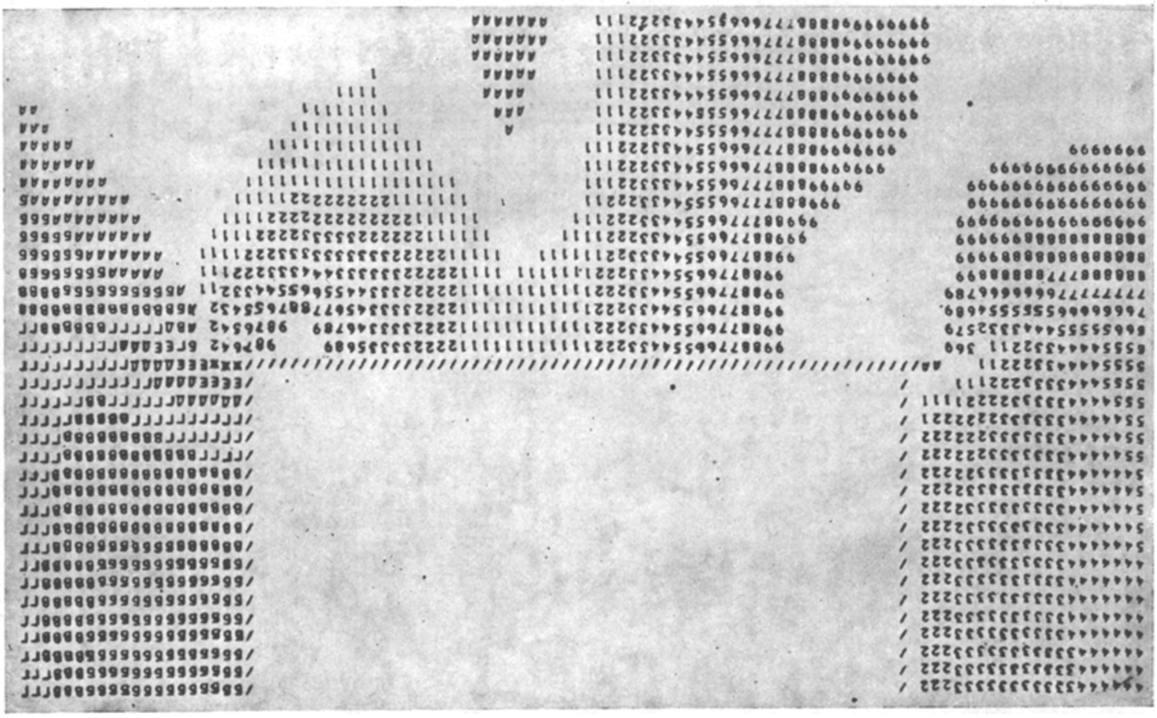

Fig. 9

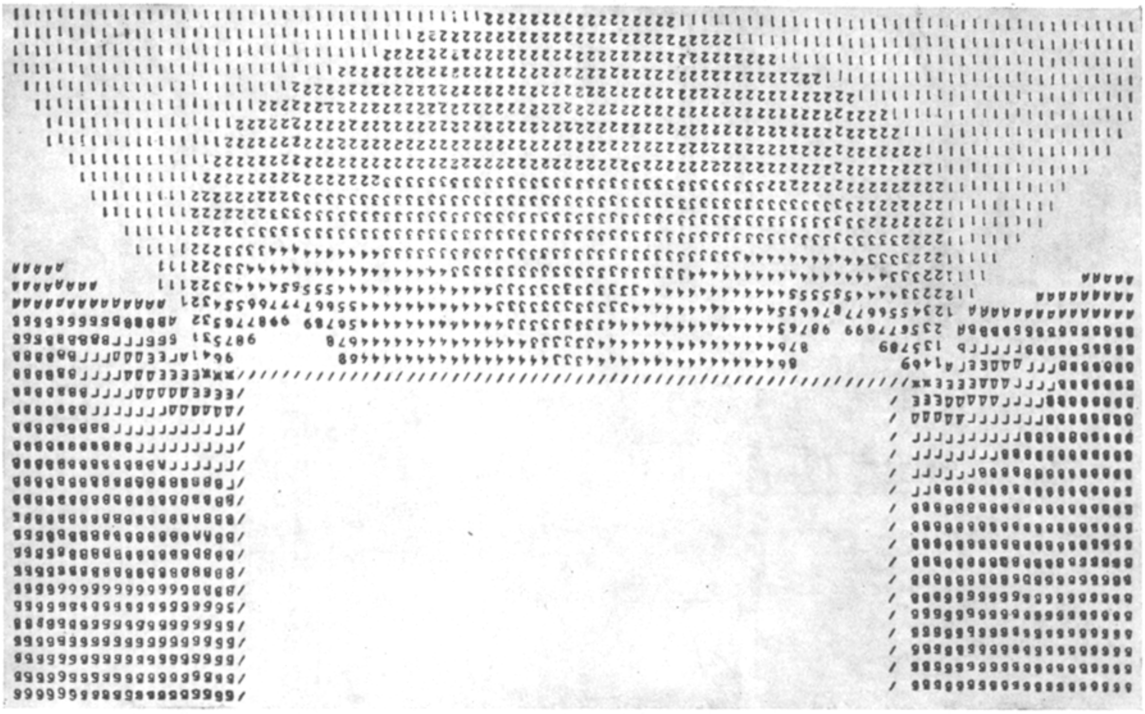

Fig. 12

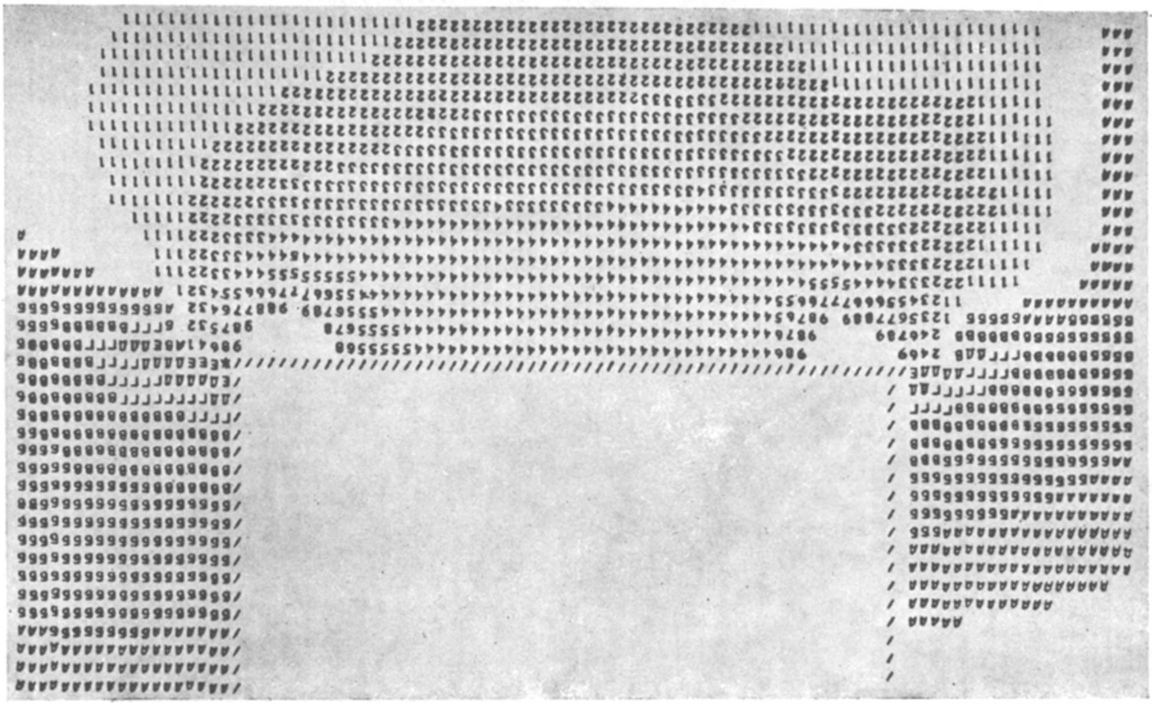

Fig. 11

It is of interest to note that, during the period of not-fully established motion (Figs. 9, 10, 11), the effects of the stress concentration in the middle part of the upper and lower bases of the cylinder is less ($\sigma_{zz} = 1.05 \pm 0.025$) than after the establishment of a quasi-steady state (Fig. 12). This difference, however, is not great, and lies within the limits 0.05 ± 0.025.

LITERATURE CITED

1. V. Vazov and J. Forsythe, Difference Methods for Solving Differential Equations in Partial Derivatives [Russian translation], Moscow, Izd. Inostr. Lit. (1963).
2. I. N. Sneddon and D. S. Berri, Classical Theory of Elasticity [Russian translation], Izd. Fizmatgiz, Moscow (1961).